Numerical study of persistence in models with absorbing states

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Extensive Monte Carlo simulations are performed in order to evaluate both the local (θ_l) and global (θ_g) persistence exponents in the Ziff-Gulari-Barshad (ZGB) [Phys. Rev. Lett. **56**, 2553 (1986)] irreversible reaction model. At the second-order irreversible phase transition (IPT) we find that both the local and the global persistence exhibit power-law behavior with a crossover between two different time regimes. On the other hand, at the ZGB first-order IPT, active sites are short lived and the persistence decays more abruptly; it is not clear whether it shows power-law behavior or not. In order to analyze universality issues, we have also studied another model with absorbing states, the contact process, and evaluated the local persistence exponent in dimensions from 1 to 4. A striking apparent *superuniversality* is reported: the local persistence exponent seems to coincide in both one- and two-dimensional systems. Some other aspects of persistence in systems with absorbing states are also analyzed.

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I. INTRODUCTION

Very many articles devoted to the study of persistence in different physical contexts have appeared recently in the literature. From a theoretical point of view, the appearance of such nontrivial critical exponents as (i) the ''nonequilibrium'' exponent λ needed to characterize the two-time correlation functions in systems relaxing in the process of quenching from infinitely high temperatures to the critical temperature T_c [1,2], and (ii) the global persistence exponent θ_g related to the probability $P_g(t) \propto t^{-\theta_g}$ that the global order parameter (e.g., the magnetization in a ferromagnet) has not changed sign up to a certain time t after a quench to T_c [3] constituted an exciting surprise. A number of theoretical, computational, and even experimental analyses were performed afterward. From the theoretical viewpoint it has been shown that, for processes where the global order parameter dynamics is Markovian, the global persistence exponent can be related to other ''traditional'' critical exponents; in particular, the following scaling law holds [3]:

$$\theta_g z = \lambda - d + 1 - \frac{\eta}{2}, \tag{1}$$

where d is the dimensionality and η is the static critical exponent of the order parameter correlation function. Nevertheless, the required hypothesis is not typically satisfied for most of the statistical models usually studied; i.e., the dynamics of the global order parameter can be argued to be generically non-Markovian. In these more generic cases Eq. (1) does not hold and θ_g is an independent nontrivial exponent [3]. In general, persistence exponents depend on the system evolution as a whole and, therefore, analytical estimations of them are scarce and difficult [4–6]. One important theoretical contribution is by Majumdar and Sire, who proposed a method to calculate autocorrelation functions perturbatively for non-Markovian processes (perturbing around a Gaussian and Markovian process [7,8]).

Recently, other similar nontrivial exponents have been uncovered and measured in different systems. Some of them are associated with the following magnitudes.

(i) The local persistence probability $P_l \propto t^{-\theta_l}$, defined as the probability that the local order parameter at a given point $\vec{x}$ has never changed sign (e.g., the probability that one spin has never been flipped in a ferromagnet) [3,9]. Local persistence probabilities have been measured in real systems such as liquid crystals and soap bubbles (see [7] and references therein).

(ii) The block persistence probability $P_L \propto t^{-\theta_L}$, defined as the probability that the order parameter integrated over a block of linear size L has never changed sign since the initial time in a phase-ordering process at finite temperature [10,11].

(iii) The generalized persistence probabilities and exponents introduced by Dornic and Godréche [12,6].

(iv) The persistence probability for domains, found first by Krapivsky and Ben-Naim in Ising systems, and subsequently in diffusion by Majumdar and Cornell [13]. These can also be generalized to other different types of pattern: the persistence probability of each pattern decays with a different exponent, a single spin or the global spin being just two limiting cases.

(v) The persistence with partial survival first studied by Majumdar and Bray and subsequently by other authors [14]. Recently it has generated a lot of interest in the context of inelastic collapse in granular materials.

For reviews of persistence studies in reversible systems see, for instance, [11,12,15].

Even though considerable effort has been devoted to the study of persistence in various models exhibiting reversible phase transitions (see [1–4,9–11,16–20] and references therein) as well as in diffusion-reaction systems (see [21,22] and references therein), far less numerous are the studies of

persistence in systems exhibiting irreversible phase transitions (IPT's) and, in particular, in systems presenting a critical point separating an active from an absorbing phase (some exceptions are [23–25]). In this context, recently, Hinrichsen and Koduvely [23] have performed a numerical study of the local and global persistence in $(1+1)$-dimensional directed percolation (DP) [26] (see also [27]). They found that the local persistence probability at the critical point (separating the absorbing from the active phase) decays as a power law with an exponent $\theta_l^{DP} \approx 1.5$. Also, global persistence measurements seemed to be consistent with $\theta_g^{DP} \approx 1.5$, a result that has to be taken with some caution since, on the one hand, the evaluation of θ_g^{DP} requires extensive simulations (see the criticisms of this result in [24]) and, on the other hand, in all the previously known models (mainly for equilibrium critical systems) $\theta_g < \theta_l$ [3]. Furthermore, it was conjectured that both θ_g and θ_l are indeed universal exponents characterizing the DP universality class. Another interesting point is that persistence exponents in this case seem to be independent of the initial condition, at odds with what happens in other well known cases such as, for example, the two-dimensional Ising model with Glauber dynamics [5]. Finally, Hinrichsen and Antoni succeeded in relating persistence exponents in directed percolation to certain ''return'' probabilities with an absorbing boundary or an active source [20]. Another important contribution to the study of persistence in IPT was presented in the recent paper by Oerding and van Wijland, in which the DP global persistence exponent was calculated analytically by combining the perturbative method developed by Majumdar and Sire [8] with standard renormalization group techniques [24].

In order to contribute further to the understanding of persistence in systems with an IPT and shed some light on the aforementioned issues, the present article is devoted to the numerical study and evaluation of persistence exponents of other models with absorbing states; in particular, the Ziff-Gulari-Barshad (ZGB) model for the catalytic oxidation of CO [28], as well as contact process [26,29]. The ZGB model (defined in two dimensions) has a twofold advantage: (i) it exhibits a second-order IPT unambiguously placed in the DP universality class [30], and (ii) it also exhibits a first-order IPT [28] where, as expected, the system does not show scaling or universal behavior [31]. In this way, the ZGB model provides a suitable framework for the study of persistence in both first- and second-order irreversible critical points. Furthermore, the present study extends the investigation of persistence exponents to higher dimensions in IPT's, by performing numerical studies of the contact process in spatial dimensions from 1 to 4.

The article is organized as follows. In Sec. II the ZGB model and the simulation method are described in detail. In Sec. III we define the local and global persistence probabilities in the systems under study, while Sec. IV is devoted to the presentation and discussion of the main results for both the ZGB model and the contact process. Finally, in Sec. V the main conclusions are presented.

II. THE ZGB MODEL AND THE SIMULATION METHOD

The ZGB model mimics the catalytic oxidation of carbon monoxide on a transition metal surface [28], namely, $2\mathrm{CO} + \mathrm{O}_2 \rightarrow 2\mathrm{CO}_2$. The lattice-gas version of the ZGB model is also known as the monomer-dimer (MD) model, where $A \equiv \mathrm{CO}$ is the monomer, as it needs a single adsorption site on the surface, and $B_2 \equiv \mathrm{O}_2$ is the dimer, which is adsorbed dissociatively, and consequently requires two neighboring sites on the catalyst surface to be adsorbed. It is assumed that the monomer-dimer reaction proceeds according to the Langmuir-Hinshelwood mechanism:

$$A(g) + S \rightarrow A(a), \tag{2}$$

$$B_2(g) + 2S \rightarrow 2B(a), \tag{3}$$

$$A(a) + B(a) \rightarrow AB(g) + 2S, \tag{4}$$

where S is an empty site on the surface, while (a) and (g) refer to the adsorbed and gas phases, respectively.

The MD model uses a square lattice to represent the catalytic surface. The Monte Carlo algorithm for its simulation is as follows. (i) an A or B_2 molecule is randomly selected with relative probabilities Y_A and Y_B, respectively. These probabilities are the relative impingement rates of the two species, which are proportional to their partial pressures. Due to the normalization $Y_A + Y_B = 1$, the model is characterized by a single parameter, say Y_A. If the selected species is A, a surface site is selected at random and, if it is vacant, A is adsorbed on it [Eq. (2)]. Otherwise, if the site is occupied, the trial ends and a new molecule is selected. If the selected species is B_2, a pair of nearest-neighbor sites is randomly chosen and the molecule is adsorbed on them only if they are both vacant [Eq. (3)]. (ii) After each adsorption event, the nearest neighbors of the added molecule are examined in order to account for the reaction given by Eq. (4). If one or more than one pairs $[B(a),A(a)]$ are identified, one of them is randomly selected and removed from the surface (for more details of the MD model and the simulation technique see, e.g., [28,32]).

Interest in the MD model arises due to its rich and complex phenomenology. In fact, in two dimensions and for the asymptotic regime $(t \rightarrow \infty)$, the system reaches a stationary state whose nature depends solely on the parameter Y_A. For $Y_A \leqslant Y_{1A} \cong 0.387\,368$ $(Y_A \geqslant Y_{2A} \cong 0.525\,54)$ the surface becomes irreversibly poisoned with B (A) particles. On the other hand, for $Y_{1A} < Y_A < Y_{2A}$ a steady state with sustained production of AB is reached. Figure 1 shows plots of the rate of AB production (R_{AB}) and the surface coverages of A particles, θ_A, and B particles, θ_B, as functions of Y_A.

At Y_{1A} and Y_{2A} the MD model exhibits irreversible phase transitions between the reactive regime and poisoned states, which are of second- and first-order, respectively. The second-order IPT belongs to the universality class of directed percolation [26] and it is rather well understood [30].

Furthermore, as shown in Fig. 1, when Y_A increases toward Y_{2A} the catalytic activity increases, and when Y_{2A} is reached a discontinuous, first-order transition appears. Above this value the activity is zero (A clusters covering the whole lattice can be observed to emerge suddenly at Y_{2A}, poisoning the system).

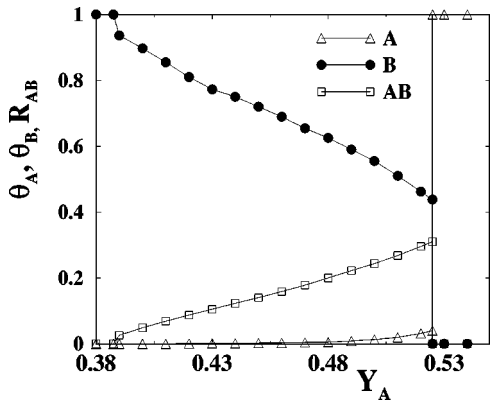

FIG. 1. Average coverage of B species (θ_B) and A species (θ_A) and the AB production rate (R_{AB}), respectively, obtained under steady state operation, as a function of Y_A for the ZGB model. Irreversible transitions of second- and first-order occur at $Y_{1A} \simeq 0.387\,368$ and $Y_{2A} \simeq 0.525\,54$, respectively.

III. LOCAL AND GLOBAL PERSISTENCE

In a system such as directed percolation, possessing absorbing states, the *local persistence* probability can be measured by starting the system with a homogeneous random initial condition, and evaluating the probability that a given, originally inactive, site has not become active up to time t [20]. It should be noted that this quantity depends upon the whole system evolution up to time t, and therefore it can be viewed as an infinite-point correlation function in the time direction. Consequently $P_l(t)$ is a rather nontrivial quantity, difficult to study analytically as discussed before. Numerical simulations in a $(1+1)$-dimensional DP model show that $P_l(t)$ decays algebraically at criticality:

$$P_l(t) \propto t^{-\theta_l} \tag{5}$$

with $\theta_l \approx 1.5$ [20]. In the case of the ZGB model, and for the continuous phase transition, the absorbing state corresponds to the surface of the catalyst fully covered by B species. This species plays the role of the inactive sites. For the other transition, i.e., the first-order one, the roles of the A and B particles are exchanged. We define $P_l^{ZGB}(t)$ as the probability that a given inactive site does not become active up to time t. Consequently, for the second-order (first-order) transition simulations are started with lattices partially covered at random by A species (B species) $\theta_{A(B)}(t=0)$. The only mechanism able to activate a site is the reaction $A+B \rightarrow AB$ as specified by Eq. (4).

We have evaluated numerically the persistence probability distribution $D_l^{ZGB}(t)$, that is, the probability for a persistent site to become active in the time interval between t and $t+dt$. One has

$$D_l^{ZGB}(t) \propto t^{-(\theta_l^{ZGB}+1)}, \tag{6}$$

where θ_l^{ZGB} is the persistence exponent [note that $D_l^{ZGB}(t)$ is simply the time derivative of P_l^{ZGB}].

In systems exhibiting reversible phase transitions, the *global persistence* is usually defined as the probability that the global order parameter (e.g., the total magnetization in the Ising model) does not change sign up to time t. In contrast, for systems with absorbing states, such as DP, the global order parameter given by the density of active sites $\rho_{AS}(t)$ is a strictly positive quantity and therefore the standard definition is not applicable. In [20] it was proposed that, instead, one can evaluate the probability that the deviation of the order parameter from its mean value, $\Delta\rho_{AS}(t) = \rho_{AS}(t) - \langle \rho_{AS}(t) \rangle$, does not change sign up to time t. In the present study, and close to the second-order IPT, we have considered

$$\rho_{AS}(t) = 1 - \theta_B = \theta_A + \theta_V, \tag{7}$$

where θ_V, θ_A, and θ_B are the densities of empty sites and those occupied by A and B species, respectively. However, in finite systems such as directed percolation the deviation probability depends on the sign of $\Delta\rho_{AS}(t)$. This asymmetry should vanish in the thermodynamic limit where $\Delta\rho_{AS}(t)$ becomes a Gaussian process. In finite systems, however, the renormalized variance of the fluctuations is no longer constant but increases with the value of $\Delta\rho_{AS}(t)$, causing the effective time scales for positive and negative fluctuations to be different [20,24]. Therefore, $P_g^{(-)}(t)$ [$P_g^{(+)}(t)$] is defined as the probability of $\Delta\rho_{AS}(t)$ to remain negative (positive) from an initial time t_{in} up to time t. We also define the probability distribution function $D_g^{-(+)}(t)$, i.e., the probability that $\Delta\rho_{AS}(t)$ changes sign in the time interval between t and $t+dt$.

In order to evaluate $\Delta\rho_{AS}(t)$ we have first to calculate a ''calibration curve'' given by $\rho_{AS} \propto t^{-\theta}$ where $\theta \simeq 0.4505$ is the corresponding DP exponent in $(2+1)$ dimensions [26,33], describing the time decay of a homogeneous initial condition at criticality: every time $\rho_{AS}(t)$ intersects this curve, $\Delta\rho_{AS}(t)$ changes sign.

A second important issue related to the global persistence exponent, worth discussion before proceeding, is the time regime in which it should be measured. Following [24], the global exponent is well defined in the regime in which the dependence on the initial state has been erased. On the other hand, there is also an upper bound to the validity of measurements given by the system-size upper cutoff. Putting these two constraints together we obtain [24]

$$L^z \gg t \gg \theta_B^{2z/(d-\eta)} \tag{8}$$

(for general systems with absorbing states, one has to substitute for θ_B the initial density of active sites). Using the DP values for the exponents, this implies $18\,300 \gg t \gg 1$ for system size $L=256$, and $65\,000 \gg t \gg 1$ for $L=512$. Therefore, as a safe, conservative limit, we start measuring at $t_{in}=100$.

It is worth stressing that in order to obtain reliable data for persistence exponents one has to perform extensive simulations. The evaluation of the local persistence requires long runs (up to 10^4 Monte Carlo steps), and reliable statistics are obtained by averaging over 10^4 (2.5×10^3) different initial configurations for $L=128$ ($L=256$). On the other hand, each single point for the global persistence distribution requires a whole run; therefore, good statistics are much tougher to

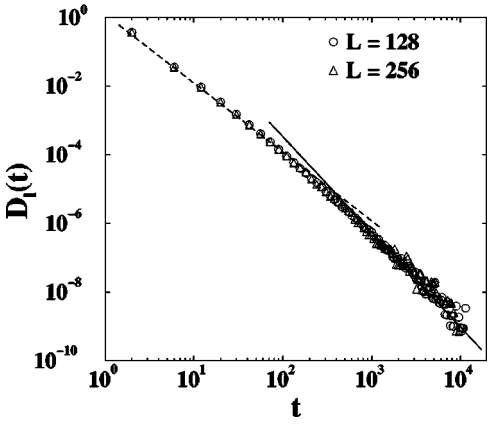

FIG. 2. Log-log plot of $D_l^{ZGB}(t)$ versus t at the second-order IPT (for lattices of linear size $L=128$, ○, and $L=256$, △ respectively). The dashed (full) line shows the short- (long-) time regime with a slope $-(\theta_l+1)=-2.00$ ($-\theta_l^{ZGB}-1=-2.5$). Results are obtained starting simulations with $\theta_B(t=0)=0.1$ and averaging over 10^4 (2.5×10^3) realizations for $L=128$ ($L=256$).

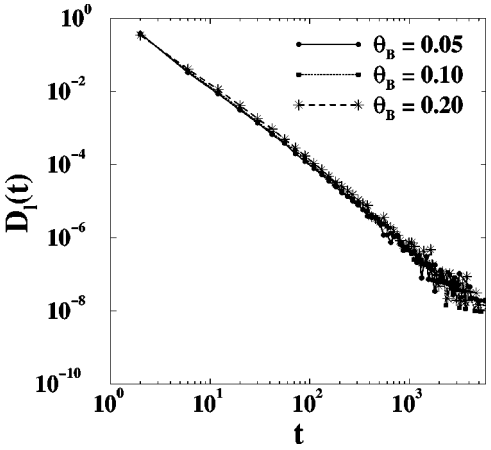

FIG. 3. Log-log plot of $D_l^{ZGB}(t)$ versus t at the second-order IPT for lattices of size $L=256$. Results obtained starting simulations with different values of $\theta_B(t=0)$ as indicated in the figure, and averaging over 2.5×10^3 realizations. No significant modification in the slope is observed upon changing the initial condition.

obtain than for local persistence. We averaged over 10^6 and 2×10^5 different runs for $L=256$ and $L=512$, respectively.

IV. RESULTS AND DISCUSSION

In this section we report our main numerical findings. The value of the second-order critical point of the ZGB model reported previously, $Y_A=0.387\,368$ [30], corresponds to the infinite-size limit; while for the system sizes we will consider, namely, 128^2 and 256^2, finite-size corrections shift the critical value to slightly larger values: in particular, all the simulations at the critical point reported in what follows are performed at $Y_A=0.3907$ [30].

Figure 2 shows a log-log plot of $D_l(t)$ versus t at the second-order IPT. Two regimes can be observed: (i) The short-time regime for $t<200$, where $D_l(t)$ exhibits a power-law behavior $D_l(t)\propto t^{-(\theta_l+1)}$ with $\theta_l\simeq1.00\pm0.05$, and (ii) the asymptotic regime for $t>200$, with the exponent $\theta_l^{ZGB}\simeq1.50\pm0.10$. Since the phase transition belongs in the DP universality class [30], it is quite surprising that the exponent obtained, within error bars, is almost the same as the one reported by Hinrichsen and Antoni [20] for a DP process in $(1+1)$ dimensions, namely, $\theta_l^{DP}\simeq1.50\pm0.03$. This finding suggests that for the DP class the difference between the persistence exponents in $(1+1)$ and $(2+1)$ dimensions is very small.

In addition, we have verified the lack of dependence of these exponents upon modifications of the initial condition, as shown in Fig. 3. In fact, runs performed using $0.05\leqslant\theta_B(t=0)\leqslant0.20$ show that neither the short-time regime nor the asymptotic regime depend on the initial density of inactive sites. This finding is in accordance with the observations in [20].

In order to gain some insight into the origin of the crossover observed in Figs. 2 and 3, we have evaluated the time dependence of the fraction of persistent sites with N_N empty nearest-neighbor sites ($N_N=0$, 1, 2, 3, and 4 respectively) as a function of time (see Fig. 4). The average number of persistent sites with N_N empty sites becomes ''almost'' vanishing at different times: $t\simeq10$ (for $N_N=0$); $t\simeq15$ (for $N_N=1$); $t\simeq30$ (for $N_N=2$); $t\simeq120$ (for $N_N=3$); and $t\simeq300$ (for $N_N=4$). Comparing the results of Figs. 2 and 3 with 4, we observe that the crossover in Figs. 2 and 3 takes place when almost all persistent sites are blocked by inactive (absorbing) ones ($t\simeq300$). The physical interpretation of the observed crossover is therefore the following. There is an initial regime in which the persistence of a given site is dominated by its open neighboring sites: particles can land directly in the neighboring sites causing desorption, and consequently the loss of persistent sites. However, as the number of such open sites is reduced on average to a value close to zero, there is a crossover to the true asymptotic regime in which one has to wait for local rearrangements in order to reach persistent sites.

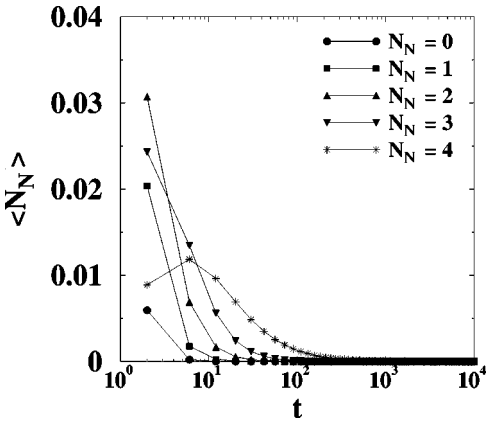

FIG. 4. Linear-logarithmic plot of the average fraction of persistent sites with N_N empty neighbors, for $N_N=0$, 1, 2, 3, and 4, respectively, as a function of t. Results are obtained using the same conditions as in Fig. 2. For times larger than 10^3 the fraction of empty sites neighboring persistent ones becomes very small, i.e., typically, activity takes place away from persistent sites, and local outbursts of activity are required in order to reach persistent sites.

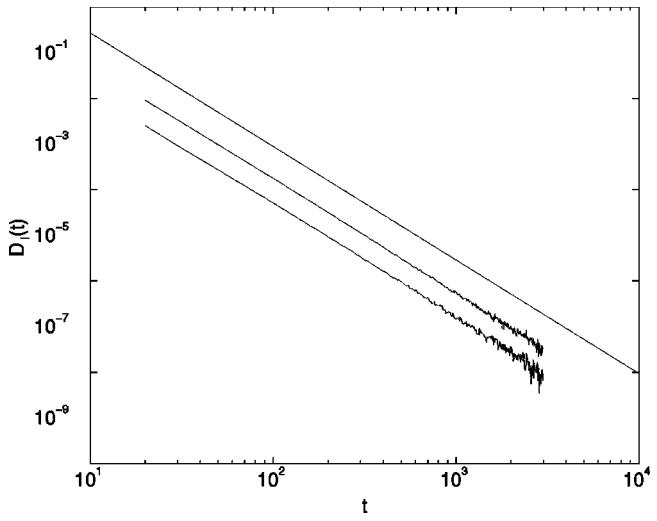

FIG. 5. Local persistence for one- and two-dimensional simulations of the contact process. In $d=1$ ($d=2$) the system size is $L=10^4$ (256^2). Observe that the two slopes are indistinguishable. The straight line has a slope $-\theta_l-1=-2.5$.

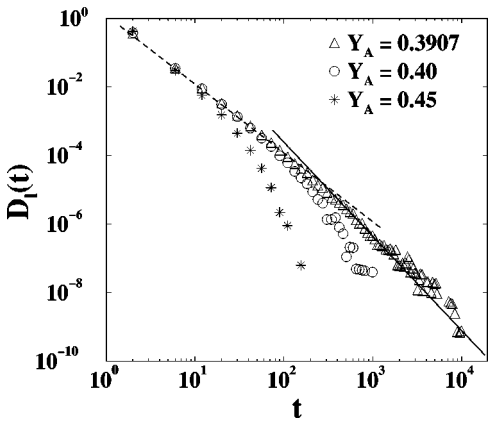

FIG. 6. Log-log plot of $D_l^{ZGB}(t)$ versus t at and away from the critical point. Results obtained using lattices of size $L=256$, with $\theta_B(t=0)=0.10$, and averaging over 2.5×10^3 realizations. Different values of the parameter Y_A were used, as indicated in the figure. The dashed (full) line shows the short- (long-) time regime with a slope -2.00 ($-\theta_l^{ZGB}-1=-2.5$), and has been drawn to allow comparison with Fig. 2. See more details in the text.

In order to further explore the dependence of the persistence exponents on the dimensionality we have also performed extensive simulations [34] of another system with absorbing states belonging to the DP universality class, namely, the contact process (CP) [26,29]. In this model, proceeding analogously as we have done for the ZGB model, we observe no crossover similar to the one described for the ZGB model. Asymptotic regimes can be obtained with less computational effort, and the exponent values are by far more accurate. We simulated the CP in dimensions ranging from 1 to 4. The largest system sizes considered were 10^4 in $d=1$, 256^2 in $d=2$, 50^3 in $d=3$, and 32^4 in $d=4$. Our results are $\theta_l\simeq1.50\pm0.01$ for $(1+1)$ and $\theta_l\simeq1.50\pm0.01$ for $(2+1)$ dimensions [34] (see Fig. 5). In fact, the two curves in Fig. 5 are strikingly parallel. This fact confirms our rather strange finding that the local persistence exponent is (almost?) the same in one- and two-dimensional systems with absorbing states. It could be the case that there is some small difference between the exponent values in $d=1$ and $d=2$, but within our numerical accuracy they are absolutely indistinguishable. The apparent coincidence between the results in one and two dimensions is quite intriguing, and so far we do not have any satisfactory explanation for this surprising result.

For the sake of completeness, and in order to test whether this apparent *superuniversality* extends also to higher dimensions, we have also measured the local persistence exponents in $d=3$ and $d=4$ as we said before. The results for θ_l are 1.33 ± 0.03 in $d=3$ and 1.15 ± 0.05 in $d=4$, indicate a diminution of the local persistence as a function of dimensionality [34].

It is well known that for equilibrium models the persistence exhibits generically a power-law behavior below the critical temperature [11,12]. In order to establish analogies with irreversible systems having absorbing states, we have also measured the local persistence away from criticality, i.e., within the reactive regime of the ZGB model, as shown in Fig. 6. For $Y_A=0.40$ (i.e., very close to the critical threshold) the power-law characteristic of the short-time regime is still observed, but the persistence curve shows a clear departure from the behavior observed at criticality. Going deeper into the reactive regime, e.g., for $Y_A=0.45$, the plot shows a marked curvature and the existence of a power-law behavior can be ruled out. On the other hand, in the absorbing phase persistence curves tend to a constant for large values of t, i.e., there is a nonvanishing asymptotic probability of persisting indefinitely. This is a direct consequence of the fact that eventually the system reaches an absorbing configuration and the dynamics is arrested. Therefore, we conclude that, unlike in reversible systems where a power-law behavior holds for a wide range below the critical temperature, in systems with absorbing states the power-law behavior of the persistence is restricted to criticality.

Respecting the global persistence, Fig. 7 shows a double logarithmic plot of $D_g^+(t)$ versus t obtained at the ZGB second-order IPT for two different lattice sizes. As in the case of the local persistence, two time regimes are observed: (i) a short-time regime ($t\leqslant300$) with a power-law behavior characterized by a slope $\simeq1.25\pm0.05$, and (ii) an asymptotic regime ($t>300$) with a power-law behavior with exponent $\theta_g^{ZGB}\simeq2.5\pm0.5$. The onset of the long-time regime depends on L as shown in Fig. 6 for $L=256$ and $L=512$; however, θ_g^{ZGB} appears to be size independent. As already discussed for the case of local persistence, the crossover between the different time regimes can be linked to the results shown in Fig. 4; again the asymptotic regime in the ZGB model starts when persistent sites are almost completely surrounded by inactive ones.

Our results for the persistence exponents, namely, $\theta_l<\theta_g$, are rather surprising at first sight, since from all previously accumulated experience, mainly for equilibrium critical systems, $\theta_l>\theta_g$ [3]. It seems that irreversible critical systems may depart from this behavior.

Assuming the dynamics of the order parameter to be Mar-

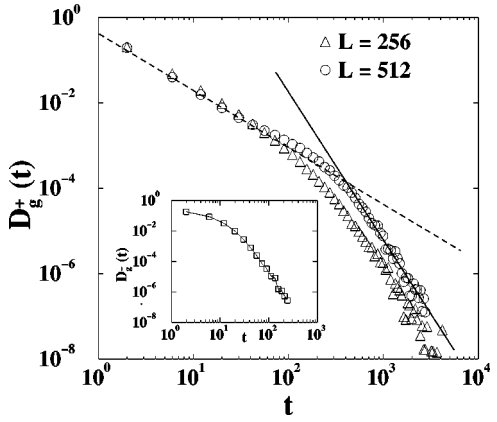

FIG. 7. Log-log plot of $D_g^+(t)$ versus t for the ZGB model at the second-order IPT for lattices of side $L=256$, △, and $L=512$, ○. The dashed (full) line shows the short- (long-) time regime and has slope -1.25 $(-\theta_g^{ZGB}-1=-3.5)$. Results have been obtained starting the simulations with $\theta_A(t=0)=0.10$ and averaging over 10^6 (2.5×10^5) realizations for $L=256$ $(L=512)$. The inset shows a log-log plot of $D_g^-(t)$ versus t, obtained for $L=512$: the asymptotic slope coincides with that of the main plot.

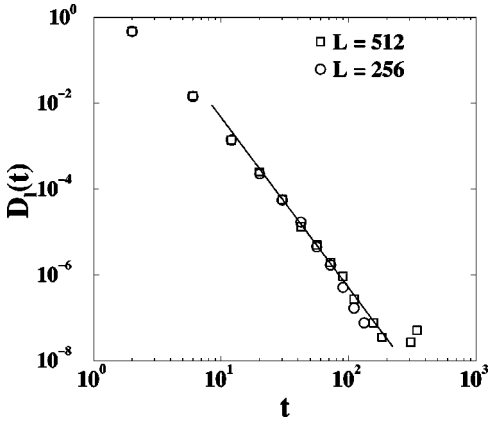

FIG. 8. Log-log plot of $D_l(t)$ versus t obtained at the coexistence point $(Y_{2A}=0.525\,54)$ for lattice sizes $L=256$ and $L=512$. The straight line has slope $-\theta_l-1=-4.00$. Results have been obtained starting simulations with $\theta_A(t=0)=0.1$ and averaging over 10^4 (2.5×10^3) realizations for $L=256$ $(L=512)$. The upward bending of the last two points indicates that the value of the critical point we are considering is slightly in the subcritical phase for the finite system sizes under analysis.

kovian, and using the known scaling relations and values of the directed percolation universality class, one obtains the result that Eq. (1) can be written as [24,33] $\theta_g=1+d/(2z)\approx1.565$ (in $d=2$). Our numerical value is far from this result, implying that the process is strongly non-Markovian. Oerding and van Wijland have shown [24], using field theoretical tools combined with the perturbative expansion method by Majumdar and Sire [3], that up to first order in ϵ expansion $\theta_g=2+0.059\epsilon+\Theta(\epsilon^2)$. This implies $\theta>2$ (at least up to first order) as, in fact, is the case in our measurements.

A qualitative physical interpretation for the fact that $\theta_l<\theta_g$ is the following: at the critical point, in the activity-density decay process, the system is more likely to fluctuate around the global average density [which corresponds to annihilating more (or less) activity than the average at that time] than invading absorbing regions in which persistent sites exist. This is very reasonable since in general, at the critical point and for large enough times, activity is restricted to some patches, which become smaller and smaller as time runs. Fluctuations in the density of these patches (an effect controlled by the global persistence exponent) are therefore more likely than invasion of persistent sites (an effect controlled by the local exponent), which are typically surrounded at large times by absorbing sites.

The inset of Fig. 7 shows a log-log plot of $D_g^-(t)$. Here the events are short lived, making difficult the evaluation of the persistence exponent. However, considering the larger error bars in this case, the exponent evaluated for $D_g^-(t)$ is consistent with the exponent previously calculated for $D_g^+(t)$. Therefore, within the discussed limitations for the evaluation of the exponents, we expect that both $D_g^-(t)$ and $D_g^+(t)$ exhibit power-law behavior with the same exponent in the large system size and time limits, in agreement with the predictions of Oerding and Van Wijland [24].

Since the ZGB model also exhibits a first-order IPT at $Y_{2A}\simeq0.525\,54$, we have evaluated the persistence at the coexistence point. Figure 8 shows a log-log plot of $D_l^{ZGB}(t)$ versus t as obtained for lattices of size $L=512$. Since the results are almost independent of L, finite-size effects may be almost negligible. Figure 8 also shows that at coexistence the persistent sites are short lived and consequently the persistence drops off abruptly. The data can be fitted by a power law with an exponent $\theta_l\simeq3.0\pm0.3$. However, since the range of the fit is narrow and there seems to be a systematic curvature, we cannot disregard an exponential (or a stretched exponential) decay for longer times. Further clarification of this issue will have to wait for computationally expensive large-scale numerical simulations. It should be noticed that the operation of a power law (scale invariance) in the dynamical critical properties of the first-order IPT of the ZGB model has recently been ruled out [31]. But given the lack of a general theoretical framework, we do not know whether or not algebraic decay of persistence functions should be expected at nonequilibrium first-order transitions.

V. CONCLUSIONS

A numerical study of persistence in the ZGB dimer-monomer model for the catalyzed reaction $A+\frac{1}{2}B_2\rightarrow AB_2$ is presented. It is found that for the second-order IPT the time dependence of both the local and the global persistence exhibits a crossover between a short- and a long-time regime. A physical explanation for such a crossover has been provided. Persistence exponents are evaluated within the long-time regime, giving $\theta_l^{ZGB}\simeq1.50\pm0.01$ for the local one and $\theta_g^{ZGB}\simeq2.5\pm0.5$ for the global one. The former, which should correspond to the DP universality class in $(2+1)$ dimensions, is very close to the value reported by Hinrichsen and Koduvely [23] for DP in $(1+1)$ dimensions, namely,

$\theta_l^{DP} \simeq 1.50 \pm 0.03$. Simulations of the contact process (another model in the same universality class) confirm this indistinguishability between the one- and two-dimensional local persistence exponents with higher accuracy (see the striking parallelism of the curves in Fig. 5). This finding implies a theoretical and numerical puzzle: Is θ_l^{DP} independent of the dimensionality for $d \leqslant 2$, or is it just that the one- and two-dimensional exponents incidentally take very similar numerical values? The likely possibility of them being equal is quite intriguing since the return probabilities in systems with absorbing states are expected to be rather different in different spatial dimensions. For the contact process, in dimensions larger than $d=2$ we find, as expected, dimension dependent exponents.

The fact that $\theta_g^{ZGB} > \theta_l^{ZGB}$ (in agreement with field theoretical predictions [24]) is in marked contrast to well established findings in the field of reversible transitions, where quite generically $\theta_g < \theta_l$. A physical explanation for such a discrepancy has been provided.

We have also observed that power laws for persistence are obtained only at criticality, and the associated exponents do not depend upon the considered initial conditions, unlike what is known for other well known reversible systems.

Persistence probabilities have also been evaluated at the first-order IPT of the ZGB model. Due to the short-lived behavior of the persistent sites we obtained an exponent $\theta_l \simeq 3.0 \pm 0.3$. However, further effort will be required in order to clarify if this apparent power-law behavior reflects merely a short-time regime followed by a cutoff or if it is a real asymptotic effect.

We expect our numerical findings to constitute a valuable step for the development of a theoretical framework in the field of irreversible phase transitions, which is certainly needed.

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